RECOGNISING ACHIEVEMENT

ADVANCED GCE **4763/01**

MATHEMATICS (MEI)

Mechanics 3

FRIDAY 23 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

1 **(a)** **(i)** Write down the dimensions of velocity, acceleration and force. [3]

A ball of mass m is thrown vertically upwards with initial velocity U. When the velocity of the ball is v, it experiences a force λv^2 due to air resistance where λ is a constant.

(ii) Find the dimensions of λ. [2]

A formula approximating the greatest height H reached by the ball is

$$H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2}$$

where g is the acceleration due to gravity.

(iii) Show that this formula is dimensionally consistent. [4]

A better approximation has the form $H \approx \dfrac{U^2}{2g} - \dfrac{\lambda U^4}{4mg^2} + \frac{1}{6}\lambda^2 U^\alpha m^\beta g^\gamma$.

(iv) Use dimensional analysis to find α, β and γ. [5]

(b) A girl of mass 50 kg is practising for a bungee jump. She is connected to a fixed point O by a light elastic rope with natural length 24 m and modulus of elasticity 2060 N. At one instant she is 30 m vertically below O and is moving vertically upwards with speed $12\,\mathrm{m\,s^{-1}}$. She comes to rest instantaneously, with the rope slack, at the point A. Find the distance OA. [4]

2 A particle P of mass 0.3 kg is connected to a fixed point O by a light inextensible string of length 4.2 m.

Firstly, P is moving in a horizontal circle as a conical pendulum, with the string making a constant angle with the vertical. The tension in the string is 3.92 N.

(i) Find the angle which the string makes with the vertical. [2]

(ii) Find the speed of P. [4]

P now moves in part of a vertical circle with centre O and radius 4.2 m. When the string makes an angle θ with the downward vertical, the speed of P is $v\,\mathrm{m\,s^{-1}}$ (see Fig. 2). You are given that $v = 8.4$ when $\theta = 60°$.

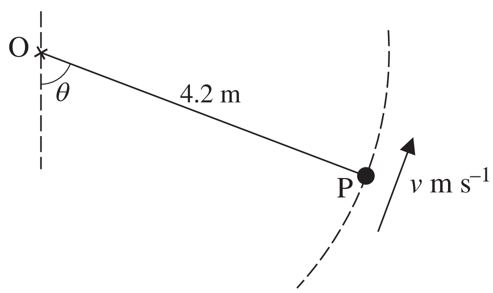

Fig. 2

(iii) Find the tension in the string when $\theta = 60°$. [3]

(iv) Show that $v^2 = 29.4 + 82.32\cos\theta$. [4]

(v) Find θ at the instant when the string becomes slack. [5]

3 A small block B has mass 2.5 kg. A light elastic string connects B to a fixed point P, and a second light elastic string connects B to a fixed point Q, which is 6.5 m vertically below P.

The string PB has natural length 3.2 m and stiffness $35\,\text{N m}^{-1}$; the string BQ has natural length 1.8 m and stiffness $5\,\text{N m}^{-1}$.

The block B is released from rest in the position 4.4 m vertically below P. You are given that B performs simple harmonic motion along part of the line PQ, and that both strings remain taut throughout the motion. Air resistance may be neglected. At time t seconds after release, the length of the string PB is x metres (see Fig. 3).

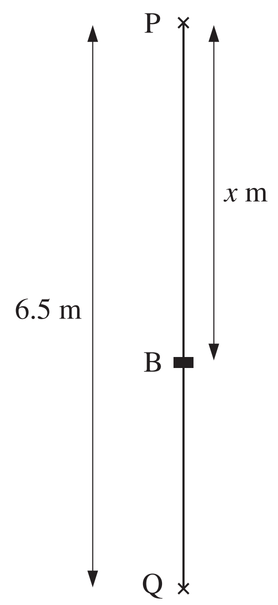

Fig. 3

(i) Find, in terms of x, the tension in the string PB and the tension in the string BQ. [3]

(ii) Show that $\dfrac{d^2x}{dt^2} = 64 - 16x$. [4]

(iii) Find the value of x when B is at the centre of oscillation. [2]

(iv) Find the period of oscillation. [2]

(v) Write down the amplitude of the motion and find the maximum speed of B. [3]

(vi) Find the time after release when B is first moving *downwards* with speed $0.9\,\text{m s}^{-1}$. [4]

[Question 4 is printed overleaf.]

4 **(a)** A uniform solid of revolution is obtained by rotating through 2π radians about the y-axis the region bounded by the curve $y = 8 - 2x^2$ for $0 \leqslant x \leqslant 2$, the x-axis and the y-axis.

(i) Find the y-coordinate of the centre of mass of this solid. [7]

The solid is now placed on a rough plane inclined at an angle θ to the horizontal. It rests in equilibrium with its circular face in contact with the plane as shown in Fig. 4.

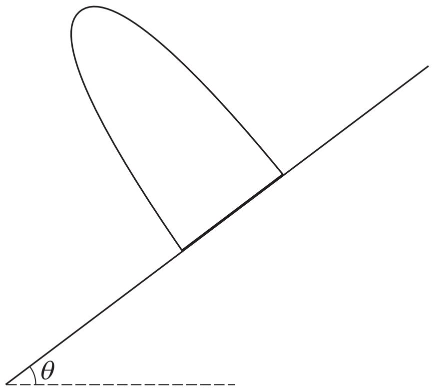

Fig. 4

(ii) Given that the solid is on the point of toppling, find θ. [4]

(b) Find the y-coordinate of the centre of mass of a uniform lamina in the shape of the region bounded by the curve $y = 8 - 2x^2$ for $-2 \leqslant x \leqslant 2$, and the x-axis. [7]

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1(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*(Deduct 1 mark if kg, m, s are consistently used instead of M, L, T)*
(ii)	$[\,\lambda\,] = \dfrac{[\,\text{Force}\,]}{[\,v^2\,]} = \dfrac{\mathrm{M\,L\,T^{-2}}}{(\mathrm{L\,T^{-1}})^2}$ $= \mathrm{M\,L^{-1}}$	M1 A1 cao **2**	
(iii)	$\left[\dfrac{U^2}{2g}\right] = \dfrac{(\mathrm{L\,T^{-1}})^2}{\mathrm{L\,T^{-2}}} = \mathrm{L}$ $\left[\dfrac{\lambda U^4}{4mg^2}\right] = \dfrac{(\mathrm{M\,L^{-1}})(\mathrm{L\,T^{-1}})^4}{\mathrm{M}(\mathrm{L\,T^{-2}})^2}$ $= \dfrac{\mathrm{M\,L^3\,T^{-4}}}{\mathrm{M\,L^2\,T^{-4}}} = \mathrm{L}$ [*H*] = L ; all 3 terms have the same dimensions	B1 cao M1 A1 cao E1 **4**	*(Condone constants left in)* *Dependent on B1M1A1*
(iv)	$(\mathrm{M\,L^{-1}})^2\,(\mathrm{L\,T^{-1}})^{\alpha}\,\mathrm{M}^{\beta}\,(\mathrm{L\,T^{-2}})^{\gamma} = \mathrm{L}$ $\beta = -2$ $-2+\alpha+\gamma = 1$ $-\alpha - 2\gamma = 0$ $\alpha = 6$ $\gamma = -3$	B1 cao M1 A1 A1 cao A1 cao **5**	At least one equation in α, γ One equation correct

(b)	EE is $\frac{1}{2}\times\frac{2060}{24}\times 6^2 \quad (=1545)$ (PE gained) = (EE lost) + (KE lost) $50\times 9.8\times h = 1545+\frac{1}{2}\times 50\times 12^2$ $490h = 1545+3600$ $h = 10.5$ OA $= 30 - h = 19.5$ m	B1 M1 F1 A1 **4**	 Equation involving PE, EE and KE Can be awarded from start to point where string becomes slack *or* any complete method (e.g. SHM) for finding v^2 at natural length If B0, give A1 for $v^2 = 88.2$ correctly obtained *or* $0 = 88.2 - 2\times 9.8\times s \quad (s = 4.5)$ *Notes* $\frac{1}{2}\times\frac{2060}{24}\times 6$ *used as EE can earn B0M1F1A0* $\frac{2060}{24}\times 6$ *used as EE gets B0M0*

2 (i)	$T\cos\alpha = mg$ $3.92\cos\alpha = 0.3\times 9.8$ $\cos\alpha = 0.75$ Angle is $41.4°$ (0.723 rad)	M1 A1 **2**	Resolving vertically *(Condone sin / cos mix for M marks throughout this question)*
(ii)	$T\sin\alpha = m\dfrac{v^2}{r}$ $3.92\sin\alpha = 0.3\times\dfrac{v^2}{4.2\sin\alpha}$ Speed is 4.9 m s^{-1}	M1 B1 A1 A1 **4**	Force and acceleration towards centre (condone $v^2/4.2$ or $4.2\omega^2$) For radius is $4.2\sin\alpha$ $(=2.778)$ Not awarded for equation in ω unless $v=(4.2\sin\alpha)\omega$ also appears
(iii)	$T - mg\cos\theta = m\dfrac{v^2}{a}$ $T - 0.3\times 9.8\times\cos 60° = 0.3\times\dfrac{8.4^2}{4.2}$ Tension is 6.51 N	M1 A1 A1 **3**	Forces and acceleration towards O
(iv)	$\frac{1}{2}mv^2 - mg\times 4.2\cos\theta = \frac{1}{2}m\times 8.4^2 - mg\times 4.2\cos 60°$ $v^2 - 82.32\cos\theta = 70.56 - 41.16$ $v^2 = 29.4 + 82.32\cos\theta$	M1 M1 A1 E1 **4**	For $(-)mg\times 4.2\cos\theta$ in PE Equation involving $\frac{1}{2}mv^2$ and PE
(v)	$(T) - mg\cos\theta = m\dfrac{v^2}{a}$ $(T) - m\times 9.8\cos\theta = m\times\dfrac{29.4+82.32\cos\theta}{4.2}$ String becomes slack when $T=0$ $-9.8\cos\theta = 7 + 19.6\cos\theta$ $\cos\theta = -\dfrac{7}{29.4}$ $\theta = 104°$ (1.81 rad)	M1 M1 A1 M1 A1 **5**	Force and acceleration towards O Substituting for v^2 *Dependent on first M1* *No marks for* $v=0 \Rightarrow \theta = 111°$

3 (i)	$T_{PB} = 35(x-3.2) \quad [= 35x - 112]$ $T_{BQ} = 5(6.5 - x - 1.8)$ $= 5(4.7 - x) \quad [= 23.5 - 5x]$	B1 M1 A1 **3**	Finding extension of BQ
(ii)	$T_{BQ} + mg - T_{PB} = m\dfrac{d^2x}{dt^2}$ $5(4.7-x) + 2.5\times 9.8 - 35(x-3.2) = 2.5\dfrac{d^2x}{dt^2}$ $160 - 40x = 2.5\dfrac{d^2x}{dt^2}$ $\dfrac{d^2x}{dt^2} = 64 - 16x$	M1 A2 E1 **4**	Equation of motion (condone one missing force) Give A1 for three terms correct
(iii)	At the centre, $\dfrac{d^2x}{dt^2} = 0$ $x = 4$	M1 A1 **2**	
(iv)	$\omega^2 = 16$ Period is $\dfrac{2\pi}{\sqrt{16}} = \frac{1}{2}\pi = 1.57$ s	M1 A1 **2**	Seen or implied *(Allow M1 for* $\omega = 16$*)* Accept $\frac{1}{2}\pi$
(v)	Amplitude $A = 4.4 - 4 = 0.4$ m Maximum speed is $A\omega$ $= 0.4 \times 4 = 1.6 \text{ m s}^{-1}$	B1 ft M1 A1 cao **3**	ft is $\lvert 4.4 - \text{(iii)} \rvert$
(vi)	$x = 4 + 0.4\cos 4t$ $v = (-)\,1.6\sin 4t$ When $v = 0.9$, $\sin 4t = -\dfrac{0.9}{1.6}$ $4t = \pi + 0.5974$ Time is 0.935 s	M1 A1 M1 A1 cao **4**	For $v = C\sin\omega t$ or $C\cos\omega t$ *This M1A1 can be earned in (v)* Fully correct method for finding the required time e.g. $\frac{1}{4}\arcsin\dfrac{0.9}{1.6} + \frac{1}{2}\text{period}$
	OR $0.9^2 = 16(0.4^2 - y^2)$ $y = -0.3307$ M1 $y = 0.4\cos 4t$ A1 $\cos 4t = -\dfrac{0.3307}{0.4}$ $4t = \pi + 0.5974$ M1 Time is 0.935 s A1 cao		Using $v^2 = \omega^2(A^2 - y^2)$ *and* $y = A\cos\omega t$ or $A\sin\omega t$ For $y = (\pm)\,0.331$ *and* $y = 0.4\cos 4t$

4 (a)(i)	$V=\int \pi x^2\,\mathrm{d}y \;=\int_0^8 \pi(4-\frac{1}{2}y)\,\mathrm{d}y$ $=\pi\left[4y-\frac{1}{4}y^2\right]_0^8=16\pi$ $V\bar{y}=\int \pi y x^2\,\mathrm{d}y$ $=\int_0^8 \pi y(4-\frac{1}{2}y)\,\mathrm{d}y$ $=\pi\left[2y^2-\frac{1}{6}y^3\right]_0^8=\frac{128}{3}\pi$ $\bar{y}=\frac{\frac{128}{3}\pi}{16\pi}$ $=\frac{8}{3}\quad(\approx 2.67)$	M1 A1 M1 A1 A1 M1 A1 **7**	*π may be omitted throughout* Limits not required for M marks throughout this question *Dependent on M1M1*
(ii)	CM is vertically above lower corner $\tan\theta=\frac{2}{\bar{y}}=\frac{2}{8/3}\quad(=\frac{3}{4})$ $\theta=36.9°\quad(=0.6435\text{ rad})$	M1 M1 A1 A1 **4**	Trig in a triangle including θ *Dependent on previous M1* Correct expression for $\tan\theta$ or $\tan(90-\theta)$ *Notes* $\tan\theta=\frac{2}{\text{cand's }\bar{y}}$ *implies M1M1A1* $\tan\theta=\frac{\text{cand's }\bar{y}}{2}$ *implies M1M1* $\tan\theta=\frac{1}{\text{cand's }\bar{y}}$ *without further evidence is M0M0*

(b)	$A=\int_{-2}^{2}(8-2x^2)\,dx$	M1	*May use* $0\le x\le 2$ *throughout* or $(2)\int_0^8\sqrt{4-\frac{1}{2}y}\,dy$
	$=\left[\ 8x-\frac{2}{3}x^3\ \right]_{-2}^{2}=\dfrac{64}{3}$	A1	
	$A\,\bar{y}=\int_{-2}^{2}\frac{1}{2}(8-2x^2)^2\,dx$	M1	or $(2)\int_0^8 y\sqrt{4-\frac{1}{2}y}\,dy$ *(M0 if ½ is omitted)*
	$=\left[\ 32x-\frac{16}{3}x^3+\frac{2}{5}x^5\ \right]_{-2}^{2}$	M1	For $32x-\frac{16}{3}x^3+\frac{2}{5}x^5$ *Allow one error* or $-\frac{8}{3}y(4-\frac{1}{2}y)^{\frac{3}{2}}-\frac{32}{15}(4-\frac{1}{2}y)^{\frac{5}{2}}$ or $-\frac{64}{3}(4-\frac{1}{2}y)^{\frac{3}{2}}+\frac{16}{5}(4-\frac{1}{2}y)^{\frac{5}{2}}$
	$=\dfrac{1024}{15}$	A1	
	$\bar{y}=\dfrac{1024/15}{64/3}$		
	$=\dfrac{16}{5}=3.2$	M1	*Dependent on first two M1's*
		A1 **7**	

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General Comments

Most candidates for this paper were able to demonstrate a sound understanding of the topics being examined. They generally seemed to have sufficient time to complete the paper, and they presented their work clearly. They certainly found this paper to be considerably more difficult than last year's; about a quarter of the candidates scored more than 60 marks (out of 72) and about 20% scored fewer than 30 marks.

Comments on Individual Questions

1) *(Dimensional analysis and elastic energy)*
This was the best answered question, with an average mark of about 15 (out of 18).

(a)(i) Almost every candidate gave the dimensions correctly.

(a)(ii) Almost all candidates knew how to find the dimensions of λ, although there were a few slips here.

(a)(iii) This was generally well done. Just a few candidates did not seem to realise that *g* was an acceleration.

(a)(iv) The method for finding the indices was well understood, and very many candidates carried it out accurately. Most obtained $\beta = -2$ correctly, but a very common error was to equate the power of *L* to zero instead of one. Some made slips when solving the simple simultaneous equations which they had obtained.

(b) Candidates who considered energy were quite often successful, although some forgot to include the kinetic energy. Many obtained the correct vertical displacement but did not calculate the required distance OA from it. However, a surprising number of candidates did not consider energy, and there were many failed attempts involving the tension.

2) *(Circular motion)*
The average mark on this question was about 10.

(i) Almost all candidates found the angle correctly.

(ii) Most candidates found the speed correctly. Common errors were taking the radius of the circular motion to be equal to the length of the string, and trying to resolve in the direction of the string instead of horizontally.

(iii) Very many candidates found the tension correctly, although a common error was to omit the component of the weight or to include it with the wrong sign.

(iv) It was not clear to all candidates that they should consider kinetic and potential energy here, and there were many unsuccessful attempts to derive the result from the radial equation of motion. Those who did consider energy were very often successful; the given answer enabled many candidates to correct errors which they had previously made.

(v) A lot of candidates assumed that the string would become slack when the velocity is zero. Those who did consider the tension were often successful, although here it was very common for the component of the weight to have the wrong sign, presumably because candidates did not appreciate that the term $mg\cos\theta$ would be negative when θ is obtuse.

3) *(Simple harmonic motion)*
The average mark on this question was about 10.

(i) Most candidates gave the tensions correctly.

(ii) This was generally done well, and the given answer was undoubtedly very helpful to many candidates who were able to adjust signs appropriately. The most common error was to omit the weight from the equation of motion.

(iii) This was usually answered correctly, but a common error was to assume that the tensions in the two strings would be equal.

(iv) The period was found correctly by most candidates.

(v) The methods for finding the amplitude and the maximum speed were well understood.

(vi) Although most candidates knew that they should use equations such as $v = v_{\max}\sin\omega t$, errors in signs or selecting the wrong solution to a trigonometric equation usually resulted in an incorrect answer.

4) *(Centres of mass)*
The average mark for this question was about 12.

(a)(i) The method for finding the centre of mass of a solid of revolution was well known, and most candidates noticed that the revolution was about the *y*-axis and adapted the formulae appropriately. However, a very common error was to have limits of integration (with respect to *y*) 0 to 2 instead of 0 to 8.

(a)(ii) The principles involved here were well understood, although a surprising number of candidates thought that the radius of the face in contact with the plane was 1 instead of 2.

(b) Finding the centre of mass of a lamina was done rather better than the solid of revolution in part (a)(i). Some candidates lost a factor ½ but the main errors were algebraic, such as expanding $\frac{1}{2}(8-2x^2)^2$ incorrectly.